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Some relations for bodies in a canal, with an application to wave-power absorption

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By the use of Green's theorem, relations are derived for the interaction of regular waves with a body in a canal. These relations are then applied to the problem of wave-power absorption by a body in a canal; this being taken as a model of an infinite row of wave-power absorbers. Two particular shapes of wave-power absorber are studied by use of thin-ship approximation. It is shown that the efficiency of power absorption by a body in a canal may be much reduced when there is more than one travelling wave mode present in the canal.

1. Introduction

In linearized water-wave theory, relations, based on Green's theorem, for the interaction of bodies with waves have been derived by various authors. Newman (1976) systematically derived (or in some cases rederived) all these relations, for the case of a single body in regular waves, for both two- and three-dimensional situations. These relations are useful in that they are applicable to many different problems involving bodies in regular waves and simplify the analysis of such problems.

For the case of a body in a canal moving in response to regular waves, it is again possible to derive various relations by the use of Green's theorem. Thus in § 2 of this paper the boundary-value problem for a body in a canal (with vertical walls) is formulated and in § 3 Green's theorem is applied to derive a number of different relations. The relations obtained are similar to those given by Newman (1976) for problems in two and three dimensions.

In order to show the usefulness of these relations, they are applied in § 4 to the problem of wave-power absorption by a body in a canal. Use of the relations enables the maximum efficiency of such a system to be easily calculated. The body in a canal may be taken as a model of an infinite row of wave-power absorbers because of the image effects due to the canal walls.

The results of § 3 and § 4 are entirely general in that they do not depend on the particular body under consideration. In § 5 they are used to study wave-power absorption by two particular shapes of body. In order to solve these particular problems use is made of the thin-ship approximation. The results obtained illustrate the effect of the interactions between absorbers (or, alternatively, the effect of the canal walls) on the efficiency of power absorption. A comparison is also made in this section (§ 5) with the results of Budal (1977) for 'point' absorbers. For this comparison it is necessary to

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know the velocity potential of a point source in a canal. A method of deriving this velocity potential is given in the appendix of this paper.

2. Formulation

Consider a canal with vertical walls which is of width $2L$, of infinite depth and which contains a fluid of density ρ. Cartesian co-ordinates x, y, z are chosen such that $y = 0$ is the undisturbed free surface of the fluid, with the y axis vertically downwards. The x axis is taken along the length of the canal and the z axis across the canal. The walls of the canal are taken to be at $z = \pm L$ and so the fluid occupies the region $y > 0$, $|z| \leqslant L$. Assume that a body, either floating or submerged, is situated in the fluid and may move in response to a wave incident down the canal (from $x = +\infty$). The incident wave is assumed to have radian frequency ω and to be of small amplitude A.

As the fluid is water it may be assumed to be incompressible, inviscid and irrotational. This allows the fluid motion to be represented by a velocity potential Φ, which may be expressed in the form

$$\Phi(x,y,z;t) = \mathrm{Re}\{\phi(x,y,z)\,e^{i\omega t}\}, \tag{2.1}$$

where ϕ satisfies Laplace's equation

$$\nabla^2\phi = 0 \quad \text{in the fluid.} \tag{2.2}$$

As the waves are of small amplitude the linearized boundary condition to be applied on the free surface is

$$K\phi + \partial\phi/\partial y = 0 \quad \text{on} \quad y = 0. \tag{2.3}$$

Here $K = \omega^2/g$ is the wavenumber. At great depths the fluid is at rest and so

$$\nabla\phi \to 0 \quad \text{as} \quad y \to \infty. \tag{2.4}$$

On the walls of the canal the normal velocity must be zero; thus

$$\partial\phi/\partial z = 0 \quad \text{on} \quad z = \pm L. \tag{2.5}$$

Finally, on the wetted surface S_b of the body

$$\partial\phi/\partial n = u, \tag{2.6}$$

where u is the complex amplitude of the normal velocity on S_b. The unit normal vector $\mathbf{n} = (n_1, n_2, n_3)$ is directed from the body into the fluid.

The general problem to be considered here is that of the interaction between the incident wave and the body. As the problem is a linear one it may be decomposed into a *scattering* problem and a number of *radiation* problems. The scattering problem is one where the body is held fixed and subject to an incident wave, while a radiation problem is one where the body is forced to oscillate in a given mode in the absence of an incident wave. Thus (2.6) may be written as

$$\frac{\partial\phi}{\partial n} = u = i\omega \sum_{j=1}^{6} \xi_j n_j \quad \text{on } S_b, \tag{2.7}$$

where

$$\zeta_j(t) = \mathrm{Re}\{\xi_j\, e^{i\omega t}\} \quad \text{for} \quad j = 1, \ldots, 6 \tag{2.8}$$

is the displacement of the body, in the jth mode, from its equilibrium position. Here (n_1, n_2, n_3) is the unit normal and

$$n_4 = (y-y_0)\,n_3-(z-z_0)\,n_2,$$
$$n_5 = (z-z_0)\,n_1-(x-x_0)\,n_3,$$
$$n_6 = (x-x_0)\,n_2-(y-y_0)\,n_1,$$

for (x,y,z) on S_b. (x_0, y_0, z_0) represents the point about which the body is rotating. For $j = 1, 2, 3, j$ corresponds to displacements parallel to the x, y, z axes respectively; while, for $j = 4, 5, 6, j$ corresponds to rotations about axes (passing through x_0, y_0, z_0) parallel to the x, y, z axes respectively. These six modes of motion are known as surge, heave, sway, roll, yaw and pitch (for $j = 1, \ldots, 6$ respectively).

It is now possible to write the solution ϕ as

$$\phi = gA\omega^{-1}\phi_s + i\omega \sum_{j=1}^{6} \xi_j\,\phi_j, \tag{2.9}$$

where the solution to the scattering problem ϕ_s satisfies

$$\partial\phi_s/\partial n = 0 \quad \text{on } S_b, \tag{2.10}$$

and the solution to the radiation problem ϕ_j satisfies

$$\partial\phi_j/\partial n = n_j \quad \text{on } S_b \text{ for } \quad j = 1, \ldots, 6. \tag{2.11}$$

Equations (2.9)–(2.11) ensure that the boundary condition (2.7) is satisfied. Note that ϕ_s may be written as

$$\phi_s = g^{-1}A^{-1}\omega\{\phi_I + \phi_D\}, \tag{2.12}$$

where

$$\phi_I = gA\omega^{-1}\exp\{iKx - Ky\}. \tag{2.13}$$

Here ϕ_I represents the incident wave and ϕ_D the diffracted wave. From (2.10) and (2.12)

$$\partial\phi_D/\partial n = -\partial\phi_I/\partial n \quad \text{on } S_b. \tag{2.14}$$

In order to completely specify the problems, for ϕ_j and ϕ_D, it is necessary to impose radiation conditions as $x \to \pm\infty$. To simplify the analysis it is assumed that the body is symmetric with respect to the centre-plane (the plane $z = 0$) of the canal and that the body is restricted to move in only surge, heave and pitch (motions which are symmetric with respect to the plane ($z = 0$). This symmetry means that

$$\partial\phi_s/\partial z = \partial\phi_j/\partial z = 0 \quad \text{on} \quad z = 0 \quad \text{for} \quad j = 1, 2, 6. \tag{2.15}$$

To find the form of the waves propagated to $x = \pm\infty$ consider the two regions $x \geqslant X_0$ and $x \leqslant -X_0$ ($X_0 > 0$), with the body occupying the intermediate region (see figure 1). On the planes $x = \pm X_0$ the following conditions hold:

$$\partial\phi_j/\partial x = \pm U_j^{\pm}(y,z) \quad \text{for} \quad j = 1, 2, 6 \tag{2.16}$$

and

$$\partial\phi_D/\partial x = \pm U_D^{\pm}(y,z), \tag{2.17}$$

where $U_j^{\pm}$, $U_D^{\pm}$ are at present unknown. Now as ϕ_j satisfies (2.5) and (2.15) it may be expanded, for $x \geqslant X_0$, as

$$\phi_j = \sum_{n=0}^{\infty} \epsilon_n \cos\left(\frac{n\pi z}{L}\right) \phi_{jn}(x,y), \tag{2.18}$$

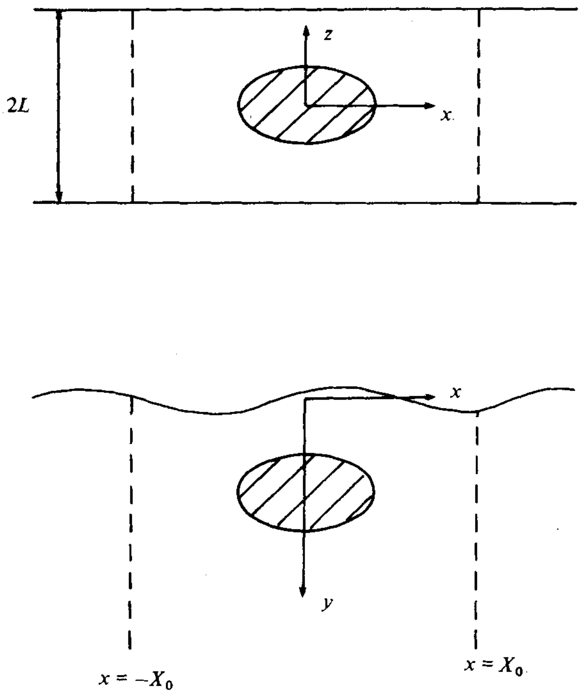


FIGURE 1. Co-ordinate system for a body in a canal.

where $\epsilon_0 = 1$ and $\epsilon_n = 2$ for $n \neq 0$. Clearly ϕ_j given by (2.18) satisfies (2.5), (2.15) and

$$\phi_{jn} = \frac{1}{2L}\int_{-L}^{L} \phi_j \cos\left(\frac{n\pi z}{L}\right) dz.$$

With this form of ϕ_j, Laplace's equation (2.2) and the free surface condition (2.3) become

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\alpha_n^2\right)\phi_{jn} = 0 \quad \text{for} \quad x \geqslant X_0 \quad \text{in the fluid,} \tag{2.19}$$

and

$$\left(K+\frac{\partial}{\partial y}\right)\phi_{jn} = 0 \quad \text{on} \quad y = 0, \tag{2.20}$$

where $\alpha_n = n\pi/L$. Equation (2.16) becomes

$$\frac{\partial \phi_{jn}}{\partial x} = U_{jn}^{+}(y) \quad \text{on} \quad x = X_0, \tag{2.21}$$

where

$$U_{jn}^{+}(y) = \frac{1}{2L}\int_{-L}^{L} U_j^{+} \cos(\alpha_n z)\, dz$$

and

$$U_j^{+}(y,z) = \sum_{n=0}^{\infty} \epsilon_n \cos(\alpha_n z)\, U_{jn}^{+}.$$

From Havelock's (1929) wave-maker theory the general solution for ϕ_{jn}, satisfying (2.19)–(2.21), may be written as

$$\phi_{jn} = A^{+}_{jn} \exp\left(-(\alpha_n^2 - K^2)^{\frac{1}{2}} x\right) \exp(-Ky) + \int_0^\infty B^{+}_{jn}(k)\,(k\cos ky - K\sin ky) \exp\left(-(\alpha_n^2 + k^2)^{\frac{1}{2}} x\right) dk, \quad (2.22)$$

where

$$\left.\begin{aligned} A^{+}_{jn} &= -2K(\alpha_n^2 - K^2)^{-\frac{1}{2}} \int_0^\infty U^{+}_{jn}(y)\, e^{-Ky}\, dy, \\ B^{+}_{jn}(k) &= -2\pi^{-1}(\alpha_n^2 + k^2)^{-\frac{1}{2}} (k^2 + K^2)^{-1} \int_0^\infty U^{+}_{jn}(y)\,(k\cos ky - K\sin ky)\, dy. \end{aligned}\right\} \quad (2.23)$$

Now if $p\pi < KL < (p+1)\pi$ for some non-negative integer p then for $n \leqslant p$ the first term in (2.22) contributes a propagating wave mode to ϕ_j as $x \to \infty$. This is because $\alpha_n^2 - K^2 < 0$ and $(\alpha_n^2 - K^2)^{\frac{1}{2}} = i(K^2 - \alpha_n^2)^{-\frac{1}{2}}$ is imaginary. If $KL = p\pi$, for $p = 1, 2, \ldots$, resonance effects occur because it is possible for a standing wave to exist across the canal (from (2.23) $A^{+}_{jp} \propto (\alpha_p^2 - K^2)^{-\frac{1}{2}}$, a square root singularity as $KL \to p\pi$).† For further discussion of this resonance see Ursell (1952); here the case $KL = p\pi$ is not studied further. For $p\pi < KL < (p+1)\pi$ there will in general exist $(p+1)$ propagating modes as $x \to +\infty$.

A similar analysis to that given above may also be applied to ϕ_j for $x \leqslant -X_0$ and to $\phi_D \geqslant X_0$ and $x \leqslant -X_0$. Thus the following results are obtained:

$$\phi_j \sim \sum_{n=0}^{p} \epsilon_n \cos(\alpha_n z)\, A^{\pm}_{jn} \exp\left(-i(K^2 - \alpha_n^2)^{\frac{1}{2}} |x| - Ky\right) \quad \text{as} \quad x \to \pm\infty, \quad (2.24)$$

$$\phi_D \sim \begin{cases} gA\omega^{-1} \sum_{n=0}^{p} \epsilon_n \cos(\alpha_n z)\, R_n \exp\left(-i(K^2 - \alpha_n^2)^{\frac{1}{2}} |x| - Ky\right) & \text{as} \quad x \to +\infty, \\ gA\omega^{-1} \sum_{n=0}^{p} \epsilon_n \cos(\alpha_n z)\, T_n \exp\left(i(K^2 - \alpha_n^2)^{\frac{1}{2}} |x| - Ky\right) & \text{as} \quad x \to -\infty, \end{cases} \quad (2.25)$$

when

$$p\pi < KL < (p+1)\pi. \quad (2.26)$$

Here R_n, T_n may be regarded as generalized reflexion and transmission coefficients for the nth propagating wave mode. Note that for $p = 0$ there is only the fundamental wave (of wavenumber K) propagating down the canal.

Although the results given above are restricted to situations symmetric with respect to the plane $z = 0$ they may easily be generalized to include the possibility of non-symmetric motions. These may be included by adding to the expression for ϕ_j in (2.18) a series of the form

$$\sum_{n=0}^{\infty} \sin\left\{\frac{(2n+1)}{2}\pi z\right\} \psi_{jn}.$$

This series satisfied condition (2.5) and so a similar analysis to that given above may be carried out. This leads to the possibility of more propagating wave modes in the canal, the number of possible modes being dependent on the value of p for which

$$\tfrac{1}{2}p\pi < KL < \tfrac{1}{2}(p+1)\pi.$$

† From (2.23) $A^{+}_{jp} = -2K(\alpha_p^2 - K^2)^{-\frac{1}{2}} \int_0^\infty U^{+}_{jp}(y)\, e^{-Ky} dy$ so that if the integral is zero or is proportional to $(\alpha_p^2 - K^2)^{\frac{1}{2}}$ there will be no singularity in A^{+}_{jp} as $KL \to \pi p$. This will probably only occur in exceptional cases.

Resonances now occur whenever $KL = \frac{1}{2}p\pi$. This generalization is not pursued here as the analysis is cumbersome and it is not required for the cases studied below.

Using the results derived above it is now possible to derive various special relations and this is carried out in the next section.

3. Derivation of relations

By applying Green's theorem to two functions ϕ, ψ which satisfy Laplace's equation (2.2) within a volume bounded by a surface S the following result may be deduced:

$$\int_S \left\{\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}\right\} dS = 0. \tag{3.1}$$

Here, use is made of this result to derive various special relations for bodies in a canal by applying it to various linear combinations of the functions ϕ_s and ϕ_j. The surface S is taken to consist of the body surface S_b, the free surface, the fluid bottom (at $y = +\infty$), the walls of the canal (at $z = \pm L$) and two vertical planes at $x = \pm X_0$ (taken such that the body lies in the region $|x| < X_0$). The results obtained are extensions of the Haskind, Newman and other relations to situations involving bodies in a canal (see Newman 1976, for analogous results for problems in two and three dimensions).

(a) *Extension of the Newman relations*

Consider the functions ϕ_s and $\phi_j - \overline{\phi}_j$ (overbar denotes complex conjugate) and apply Green's theorem to them. Note that

$$\begin{aligned} \frac{\partial}{\partial n}(\phi_j - \overline{\phi}_j) &= \frac{\partial \phi_j}{\partial n} - \frac{\partial \overline{\phi}_j}{\partial n} \\ &= (n_j - \overline{n}_j) \quad \text{by (2.11)} \\ &= 0 \quad \text{as } n_j \text{ is real.} \end{aligned}$$

This result together with (2.3), (2.4), (2.5) and (2.11) shows that the only contributions to the integral come from the vertical planes at $x = \pm X_0$. Hence, by taking the limit as $X_0 \to \infty$ and using (2.12), (2.13), (2.24) and (2.25), the following result is obtained:

$$A_{j0}^{+} + \sum_{n=0}^{p} \epsilon_n \left\{1 - \left(\frac{n\pi}{KL}\right)^2\right\}^{\frac{1}{2}} [\overline{A_{jn}^{+}} R_n + \overline{A_{jn}^{-}} T_n] = 0 \tag{3.2}$$

for $p\pi < KL < (p+1)\pi$ and $j = 1, 2, 6$. When $p = 0$

$$A_{j0}^{+} + \overline{A_{j0}^{+}} R_0 + \overline{A_{j0}^{-}} T_0 = 0,$$

which corresponds to the two-dimensional result given by Newman (1976, equation (49)). In the general case ($p \neq 0$) the extra terms arise because of the other propagating wave modes present in the canal.

(b) Extension of Haskind relations

The exciting force F_j^s acting on the body (held fixed) in the jth mode (or direction) due to an incident wave of amplitude A is given by

$$F_j^s = -\int_{S_b} p_s n_j \, dS, \tag{3.3}$$

where the pressure

$$p_s = \mathrm{Re}\{-i\rho g A \phi_s e^{i\omega t}\}; \tag{3.4}$$

from (2.11), (2.12) and (3.3)

$$F_j^s = \mathrm{Re}\left\{i\omega\rho \int_{S_b} (\phi_I + \phi_D) \frac{\partial \phi_j}{\partial n} dS \, e^{i\omega t}\right\}. \tag{3.5}$$

By applying Green's theorem to ϕ_D and ϕ_j the following result is obtained (after taking the limit $X_0 \to \infty$):

$$\int_{S_b} \left\{\phi_D \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_D}{\partial n}\right\} dS = 0.$$

Note that by (2.3)–(2.5), (2.24) and (2.25) there are no contributions to the integral from the other surfaces. By using this result together with (2.14) equation (3.5) may be written as

$$F_j^s = \mathrm{Re}\left\{i\omega\rho \, e^{i\omega t} \int_{S_b} \left\{\phi_I \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_I}{\partial n}\right\} dS\right\}.$$

Now a second application of Green's theorem (to ϕ_I and ϕ_j) allow this to be written as

$$F_j^s = \mathrm{Re}\left\{-i\omega\rho \, e^{i\omega t} \int_{S'} \left\{\phi_I \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_I}{\partial n}\right\} dS\right\},$$

where S' represents the planes at $x = \pm X_0$. Finally, by taking the limit as $X_0 \to \infty$ and using (2.13) and (2.24), the following result is obtained:

$$F_j^s = \mathrm{Re}\{\rho g A A_{j0}^+ . 2L \, e^{i\omega t}\}. \tag{3.6}$$

This result is similar to that given by Newman (1976, equation (45)) for two-dimensional problems except for the factor $2L$ (canal width). The factor $2L$ appears here as F_j^s is the total existing force on the body, whereas in the two-dimensional case F_j^s is the existing force per unit length along the body.

(c) Added-mass and damping coefficients

The radiation force F_j^r acting on the body in the jth mode due to its own motion, in the absence of an incident wave, is given by

$$F_j^r = -\int_{S_b} p_r n_j \, dS,$$

where the pressure

$$p_r = \mathrm{Re}\{-i\omega\rho \, e^{i\omega t} \sum_{k=1,2,6} i\omega \xi_k \phi_k\}.$$

Hence, using (2.8) and (2.11),

$$F_j^r = -\sum_{k=1,2,6} (M_k \ddot{\xi}_k + B_{jk} \dot{\xi}_k), \tag{3.7}$$

where

$$\omega^2 M_{jk} - i\omega B_{jk} = -\rho\omega^2 \int_{S_b} \phi_k \frac{\partial \phi_j}{\partial n} dS. \tag{3.8}$$

Here M_{jk}, B_{jk} (both real) are defined to be the added-mass and damping coefficients. By applying Green's theorem to ϕ_j, ϕ_k the following result is obtained:

$$\omega^2 M_{jk} - i\omega B_{jk} = \omega^2 M_{kj} - i\omega B_{kj}.$$

Hence, equating real and imaginary parts,

$$M_{jk} = M_{kj}, \quad B_{jk} = B_{kj}. \tag{3.9}$$

(d) *A relation between energy radiation and the damping coefficient*

Consider Green's theorem applied to $\bar{\phi}_j$ and ϕ_k. On use of (2.3)–(2.5), (2.24) and (3.8), this leads to the result

$$i\omega^2\rho \sum_{n=0}^{p} \epsilon_n \left\{1 - \left(\frac{n\pi}{KL}\right)^2\right\}^{\frac{1}{2}} [\overline{A_{jn}^+} A_{kn}^+ + \overline{A_{jn}^-} A_{kn}^-]\, 2L - [\omega^2 M_{kj} + i\omega B_{kj} - \omega^2 M_{jk} + i\omega B_{jk}] = 0.$$

Now, on use of (3.9),

$$B_{jk} = \tfrac{1}{2}\rho\omega \sum_{n=0}^{p} \epsilon_n \left\{1 - \left(\frac{n\pi}{KL}\right)^2\right\}^{\frac{1}{2}} [\overline{A_{jn}^+} A_{kn}^+ + \overline{A_{jn}^-} A_{kn}^-]\, 2L \tag{3.10}$$

for $p\pi < KL < (p+1)\pi$. For the case $p = 0$ this result is similar to the two-dimensional result (see Newman 1976, equation (31 a)) except for a factor $2L$. The reason for the appearance of the factor $2L$ is the same as the reason for its presence in the result for the exciting force F_j^s (see § 3(b) above).

(e) *Energy conservation*

Finally Green's theorem may be applied to the two functions ϕ_s and $\bar{\phi}_s$ to obtain

$$\sum_{n=0}^{p} \epsilon_n \left\{1 - \left(\frac{n\pi}{KL}\right)^2\right\}^{\frac{1}{2}} [|Rn|^2 + |T_n|^2] = 1$$

for $p\pi < KL < (p+1)\pi$. This result shows that, if the body is held fixed, the energy in the incident wave is equal to the energy in the reflected wave plus the energy in the transmitted wave. In the case $p = 0$ this result is similar to the two-dimensional result.

In the above, some new relations have been derived for a body in a canal. These will be used in the next section to study wave-power absorption by a body in a canal. The results (3.6) and (3.10) have been given previously by Evans (1979, § 3) for the case $p = 0$. In this case ($p = 0$) there is a clear correspondence between the results for a body in a canal and the results given by Newman (1976) for two-dimensional problems.

4. Wave-power absorption

Assume that the body is constrained to move in one mode only (either surge, heave or pitch) and that it can absorb power from the incident wave. In the analysis given below the maximum power that such a system can absorb will be determined. Details of the body dynamics will not be considered here; these may be studied by using the results of § 3 and by following the method used by Evans (1976) for two- and three-dimensional problems.

The power P absorbed by the body is equal to the mean rate of working of the hydrodynamic forces on the body. Thus

$$P = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} F(t)\,\dot{\zeta}_j(t)\,dt,$$

where $F(t) = F_j^s + F_j^r$ and F_j^s is given by (3.6), F_j^r by (3.7) and $\zeta_j(t)$ by (2.8). As there is motion only in the jth mode

$$F_j^r = -M_{jj}\ddot{\zeta}_j(t) - B_{jj}\dot{\zeta}_j(t).$$

Combining these results allows P to be written as

$$P = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \{F_j^s - M_{jj}\ddot{\zeta}_j - B_{jj}\dot{\zeta}_j\}\,\dot{\zeta}_j\,dt$$

$$= \tfrac{1}{2}\operatorname{Re}\{i\omega\xi_j \bar{X}_j\} - \tfrac{1}{2}\omega^2 B_{jj}|\xi_j|^2 \tag{4.1}$$

$$= |X_j|^2/8B_{jj} - \tfrac{1}{2}|i\omega\xi_j - \tfrac{1}{2}X_j/B_{jj}|^2, \tag{4.2}$$

where

$$X_j = \rho g A A_{j0}^+ 2L. \tag{4.3}$$

From (3.10) it is clear that B_{jj} is greater than or equal to zero. B_{jj} is only equal to zero if no waves are propagated to infinity by forced motions of the body in the jth mode and this will only be true for particular body shapes at particular frequencies. In general B_{jj} is therefore positive and so P is maximized by choosing

$$i\omega\xi_j = \tfrac{1}{2}X_j/B_{jj}, \tag{4.4}$$

whence

$$P_{\max} = \tfrac{1}{2}\omega^2 B_{jj}|\xi_j|^2 \quad (\xi_j \text{ given by (4.4)})$$

$$= |X_j|^2/8B_{jj}. \tag{4.5}$$

The maximum power that can be absorbed is clearly equal to the power lost by radiation from the body (see last term of (4.1)). This maximum is achieved when the velocity of the body is in phase with the exciting force (see (2.8) and (4.4)).

It is possible to define the efficiency E of this system as the ratio of the power absorbed by the body to the power available in the incident wave. Hence

$$E = P\left\{\frac{1}{4\omega}\rho g^2|A|^2 2L\right\}^{-1}. \tag{4.6}$$

It is now possible, by use of (4.3), (4.5) and (4.6), to write the maximum efficiency of the system as

$$E_{\max} = \tfrac{1}{2}\rho\omega|A_{j0}^+|^2 2L/B_{jj}$$

$$= |A_{j0}^+|^2\left\{\sum_{n=0}^{p}\epsilon_n\left[1-\left(\frac{n\pi}{KL}\right)^2\right]^{\frac{1}{2}}[|A_{jn}^+|^2 + |A_{jn}^-|^2]\right\}^{-1} \tag{4.7}$$

for $p\pi < KL < (p+1)\pi$. Here use has been made of (3.10). In the case $p = 0$,

$$E_{\max} = |A_{j0}^+|^2\{|A_{j0}^+|^2 + |A_{j0}^-|^2\}^{-1}, \tag{4.8}$$

a result proved by Evans (1979, § 3). This result is also analogous to the two-dimensional result given by Evans (1976). Equation (4.7) may be interpreted as the maximum efficiency of either a single body in a canal or an infinite array of identical absorbers. The second interpretation is possible because of the infinite set of images of the absorber in the canal walls.

For a body symmetric with respect to the y, z plane

$$|A_{jn}^{+}| = |A_{jn}^{-}| \quad \text{for} \quad n = 0, 1, \ldots, p, \tag{4.9}$$

and so

$$E_{\max} = \frac{1}{2}\left\{\sum_{n=0}^{p} \epsilon_n \left[1 - \left(\frac{n\pi}{KL}\right)^2\right]^{\frac{1}{2}} \left|\frac{A_{jn}^{+}}{A_{j0}^{+}}\right|^2\right\}^{-1}. \tag{4.10}$$

In the case $p = 0$, $E_{\max} = \frac{1}{2}$ as shown by Evans (1979). However, if $p \neq 0$, then from (4.10) it is clear that $E_{\max}$ will be less than a half. This loss of efficiency is due to the presence of other propagating wave modes, apart from the one of fundamental wavenumber K, which radiate energy away from the absorber. In general, for a non-symmetric absorber, it is clear from (4.8) that $E_{\max}$ will be greatest when $|A_{j0}^{+}|$ is large relative to $|A_{jn}^{+}|$ for $n = 1, 2, \ldots, p$ and also to $|A_{jn}^{-}|$ for $n = 0, 1, \ldots, p$. Hence a good absorber will be one which, when forced to oscillate, will generate waves of only the fundamental wavenumber K and only in the direction from which the incident wave approaches.

It is possible to prove other results for power absorption by a body in a canal. For example, by following the analysis of Srokosz & Evans (1979) for a two-dimensional problem, it is possible to prove that $E_{\max} = 1$ for $0 < KL < \pi$ (i.e. $p = 0$) if the body is allowed to move in two modes and if there is symmetry with respect to the plane $z = 0$. The details of the proof are not given here but the method works because of the similarity (noted earlier) between two-dimensional results and results for bodies in a canal when $0 < KL < \pi$. In the next section the results derived above are used to illustrate the effects of the canal walls on power absorption by the body.

5. The thin-ship approximation

In order to study the effects on $E_{\max}$ of the other propagating wave modes, apart from the one of fundamental wavenumber K, it is necessary to calculate the values of $|A_{jn}^{\pm}|$ for a particular body. In general this is difficult, but a simple problem that may be solved by using Havelock's (1929) wave-maker theory is given below.

Assume that the body is floating in the canal with its immersed shape described by $x = \pm\,\eta(y, z)$ and that it is symmetrical with respect to the planes $x = 0$ and $z = 0$. Furthermore assume that its maximum beam 2ϵ in the x direction is small compared to its length $2d$ (in the z direction, $d \leqslant L$) and its draft h (in the y direction). This is the 'thin ship' approximation, further details of which may be found in Newman (1977, pp. 305–307). Suppose that the body is forced to oscillate with unit amplitude in heave ($j = 2$): as ϵ/d, $\epsilon/h \ll 1$ it is possible to approximate in the following manner:

$$\left.\frac{\partial \phi_2}{\partial x}\right|_{x=\pm 0} = \mp \frac{\partial \eta}{\partial y} \quad \text{for} \quad |z| \leqslant d, \quad 0 \leqslant y \leqslant h. \tag{5.1}$$

Here $-\partial\eta/\partial y$ is the approximation to the vertical component of the normal body, evaluated on $\eta = 0$. By symmetry

$$\left.\frac{\partial \phi_2}{\partial x}\right|_{x=0} = 0 \tag{5.2}$$

on that part of the plane not occupied by the body. Once the body shape $x = \eta(y,z)$ is specified, equations (5.1), (5.2) and (2.10)–(2.23) allow the values of $A_{2n}^{\pm}$ (for $n = 0, 1, 2, \ldots, p$) to be determined. Here two body shapes are studied which are identical to those considered by Evans (1979) for a corresponding three-dimensional problem. It is only necessary to evaluate A_{2n}^{+} $(= A_{2n}^{-})$ because of symmetry.

Case (*a*). Body shape defined by

$$x = \eta(y,z) = \epsilon(h-y)/h \quad \text{for} \quad |z| \leqslant d, \quad 0 \leqslant y \leqslant h. \tag{5.3}$$

This body is rectangular in plan and wedge-shaped in elevation. From the results of § 2

$$A_{2n}^{+} = 2iK(K^2-\alpha_n^2)^{-\frac{1}{2}} \int_0^\infty U_{2n}^{+}(y)\, e^{-Ky}\, dy, \tag{5.4}$$

where

$$U_{2n}^{+}(y) = \frac{1}{2L}\int_{-L}^{L} U_2^{+}(y,z)\cos(\alpha_n z)\, dz \tag{5.5}$$

and

$$U_2^{+}(y,z) = \left.\frac{\partial \phi_2}{\partial x}\right|_{x=0+}. \tag{5.6}$$

From (5.1)–(5.3), (5.5) and (5.6)

$$\begin{aligned} U_{2n}^{+}(y) &= \epsilon[2Lh]^{-1}\int_{-d}^{d} \cos(\alpha_n z)\, dz \\ &= \left.\begin{cases} \epsilon d[hL]^{-1} & \text{for} \quad n = 0, \\ \epsilon[n\pi h]^{-1}\sin\left(\dfrac{n\pi d}{L}\right) & \text{for} \quad n \neq 0, \end{cases}\right\} \quad 0 \leqslant y \leqslant h, \end{aligned}$$

and

$$U_{2n}^{+}(y) = 0 \quad \text{for} \quad y > h.$$

On substitution of these results into (5.4) the following is obtained:

$$A_{2n}^{+} = i\epsilon K d h^{-1}(1-e^{-Kh}) \begin{cases} (KL)^{-1} & \text{for} \quad n = 0, \\ \left(\dfrac{n\pi d}{L}\right)^{-1} \sin\left(\dfrac{n\pi d}{L}\right)(K^2L^2 - n^2\pi^2)^{-\frac{1}{2}} & \text{for} \quad n = 1, 2, \ldots, p. \end{cases} \tag{5.7}$$

It is now possible to give an exact expression for $E_{\max}$ by substituting (5.7) into (4.10). Hence

$$E_{\max} = \frac{1}{2}\left\{1 + 2\sum_{n=1}^{p}\left(\frac{L}{n\pi d}\right)^2 \sin^2\left(\frac{n\pi d}{L}\right)\left[1-\left(\frac{n\pi}{KL}\right)^2\right]^{-\frac{1}{2}}\right\}^{-1} \tag{5.8}$$

for $p\pi < KL < (p+1)\pi$ (note that for $p = 0$ the series in (5.8) is taken to be zero and $E_{\max} = \frac{1}{2}$). There are two interesting limiting cases, $d/L \to 0$ and $d/L \to 1$. When $d/L = 1$ the body stretches across the whole width of the canal and $E_{\max} = \frac{1}{2}$; in agreement with the two-dimensional result given by Evans (1976). When $d/L \to 0$ the body becomes small and may be considered to be a 'point' absorber (cf. Budal 1977). In this case $(d/L = 0)$

$$E_{\max} = \frac{1}{2}\left\{1 + 2\sum_{n=1}^{p}\left[1-\left(\frac{n\pi}{KL}\right)^2\right]^{-\frac{1}{2}}\right\}^{-1}. \tag{5.9}$$

This result agrees with that obtained by using a point source to represent the body (see appendix).

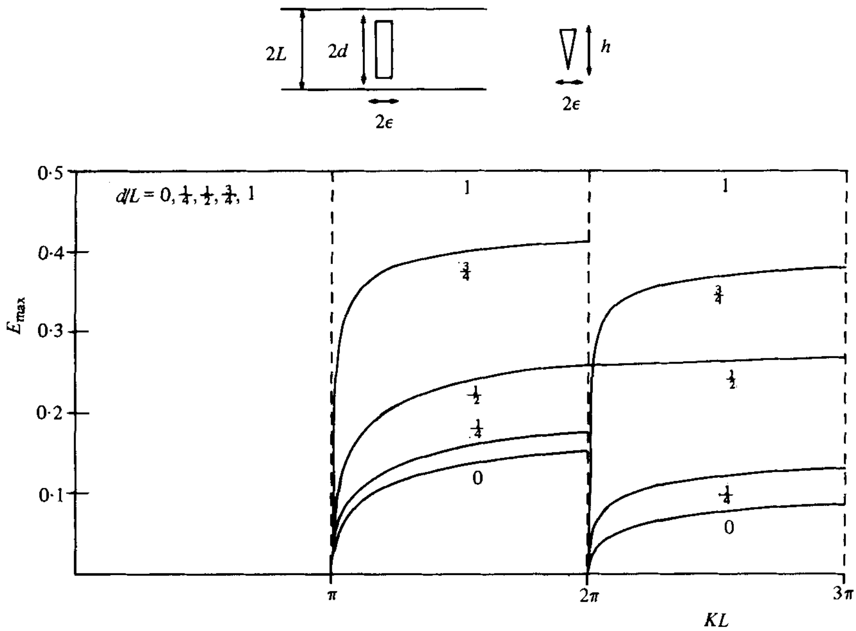


FIGURE 2. E_{max} plotted against KL for case (a) of §5.

Case (b). Body shape defined by

$$x = \eta(y,z) = \epsilon(h-y)(d^2-z^2)^{\frac{1}{2}}[hd]^{-1} \quad \text{for} \quad |z| \leqslant d, \quad 0 \leqslant y \leqslant h. \tag{5.10}$$

This body is lenticular in plan and wedge-shaped in elevation. A similar analysis to that given for case (a) above leads to the following result:

$$A_{2n}^{+} = \tfrac{1}{4} i\pi\epsilon K d h^{-1}(1-e^{-Kh}) \begin{cases} (KL)^{-1} & \text{for} \quad n = 0, \\ 2\left(\dfrac{n\pi d}{L}\right)^{-1} J_1\left(\dfrac{n\pi d}{L}\right)(K^2L^2-n^2\pi^2)^{-\frac{1}{2}} & \text{for} \quad n = 1, 2, \ldots, p. \end{cases} \tag{5.11}$$

Here use has been made of the result

$$J_1(\alpha) = \frac{2\alpha}{\pi}\int_0^1 (1-t^2)^{\frac{1}{2}} \cos(\alpha t)\, dt$$

(see Gradshteyn & Ryzhik 1965, p. 419).

The value of $E_{\max}$ obtained using (5.11) is

$$E_{\max} = \frac{1}{2}\left\{1 + 8\sum_{n=1}^{p}\left(\frac{L}{n\pi d}\right)^2 J_1^2\left(\frac{n\pi d}{L}\right)\left[1-\left(\frac{n\pi}{KL}\right)^2\right]^{-\frac{1}{2}}\right\} \tag{5.12}$$

for $p\pi < KL < (p+1)\pi$ (again if $p = 0$, the series in (5.12) is taken to be zero and $E_{\max} = \frac{1}{2}$). From (5.12) it can be seen that in the limit $d/L \rightarrow 1$, when the body occupies the whole width of the canal, $E_{\max}$ does not equal a half and so does not agree with the two-dimensional result. This is because, unlike case (a) above, the body is not of uniform cross-section along its length. The limit $d/L \rightarrow 0$ gives the result (5.9) as $J_1(\alpha) \sim \frac{1}{2}\alpha$ as $\alpha \rightarrow 0$.

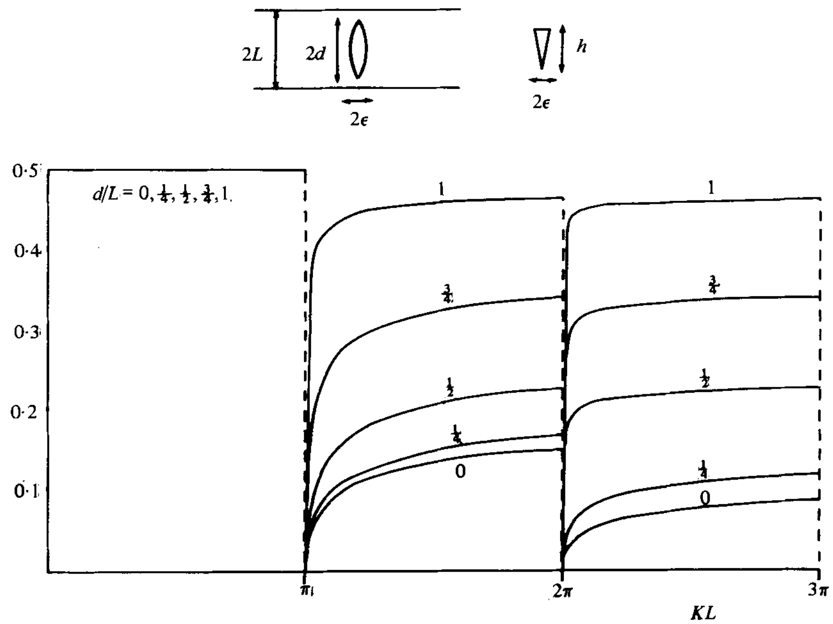


FIGURE 3. $E_{\max}$ plotted against KL for case (b) of §5.

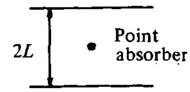
Note that the value of $E_{\max}$ in (5.8), (5.9) and (5.12) is independent of the draft, h, of the body. Varying h affects the amplitude at the waves generated but does not affect the ratio $|A_{2n}^{+}/A_{20}^{+}|$ and so does not influence $E_{\max}$.

Results and discussion of cases (a) *and* (b). In figures 2 and 3 curves of $E_{\max}$, given by (5.8) and (5.12) respectively, are plotted against KL for

$$d/L = 0, \tfrac{1}{4}, \tfrac{1}{2}, \tfrac{3}{4}, 1.$$

It is clear from these figures, and from (5.8) and (5.12), that $E_{\max}$ is in general discontinuous at $KL = p\pi$ for $p = 1, 2, \ldots$. This is due to the resonant effects that occur whenever $KL = p\pi$ and also due to the appearance of an extra propagating wave mode as KL increases from $KL < p\pi$ to $KL > p\pi$. From figure 2 it can be seen that $E_{\max}$ is not discontinuous at all when $d/L = 1$ and that when $d/L = \frac{1}{2}$ it is not discontinuous at $KL = 2\pi$. From (5.8) it can be seen that the reason for this is that in both cases $\sin(n\pi d/L) = 0$ and so the extra term added to the series as KL goes from $KL < n\pi$ to $KL > n\pi$ is zero. A similar phenomenon does not appear to occur in figure 3 for case (b). However, it is obvious from (5.12) that by choosing d/L such that $J_1(n\pi d/L) = 0$ it is possible to eliminate the discontinuity at $KL = n\pi$. In general when $KL = p\pi$ $E_{\max}$ is discontinuous and drops to zero and then begins to rise again. This shows that the resonance effects and the appearance of a new propagating wave mode are detrimental to the efficiency of power absorption (although the discontinuity in $E_{\max}$ for a particular value of $KL = p\pi$ may be eliminated by a suitable choice of d/L). A similar loss of efficiency due to resonant effects was found by Srokosz & Evans (1979) for a two-dimensional problem involving vertical barriers.

From figures 2 and 3 it can be seen that, for $KL > \pi$, as d/L increases from 0 to 1, $E_{\max}$ increases. This suggests that for a long row of equally spaced absorbers the gaps



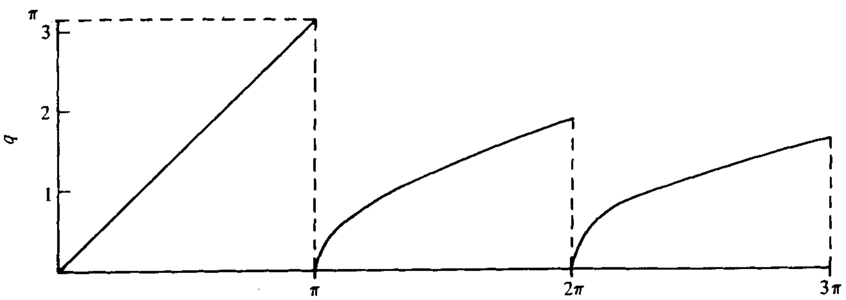


FIGURE 4. The interaction factor q plotted against KL for infinite row of point absorbers.

between the absorbers should not be too large compared to the size of the absorber. This means that small, widely spaced 'point' absorbers (as suggested by Budal 1977; Falnes & Budal 1978) will only be efficient for values of KL less than π. It should be noted that the analysis given here is for situations symmetric with respect to the plane $z = 0$. If this condition does not hold antisymmetric resonant modes are possible and so resonances occur whenever $KL = \frac{1}{2}p\pi$ ($p = 1, 2, \ldots$) and a new propagating wave mode appears when KL goes from $KL < \frac{1}{2}p\pi$ to $KL > \frac{1}{2}p\pi$. In this case therefore $E_{\max} = \frac{1}{2}$ for $KL < \frac{1}{2}\pi$, which is a stronger restriction on the values of spacing to wavelength ratios that may be chosen to give good absorption. As no realistic situation is perfectly symmetric it is likely that these antisymmetric modes will be present and they may therefore be important.

It is possible to compare the results obtained here with those of Budal (1977) for an infinite row of 'point' absorbers by considering the case $d/L = 0$ in more detail. Budal's assumptions about point absorbers are equivalent to modelling each absorber by a point source and it is shown in the appendix how this leads to the result (5.9) for $E_{\max}$ which corresponds to choosing $d/L = 0$ in cases (a) and (b) above. Budal's results are given in terms of the absorption length, which is equal to the power absorbed by the body divided by the power available per unit crest length of incident wave. For a body in a canal the maximum absorption length, $l_{\max}$, is given by

$$l_{\max} = E_{\max} \times 2L; \tag{5.13}$$

Budal (1977) writes his maximum absorption length as

$$l_{\max} = \frac{\lambda}{2\pi} q, \tag{5.14}$$

where q is an interaction factor (here $\lambda/2\pi$ is the absorption length for a single, heaving, axisymmetric absorber in three dimensions). Combining (5.13) and (5.14) gives

$$q = (2KL) \times E_{\max} \tag{5.15}$$

as $K = 2\pi/\lambda$. Now by plotting q against KL, using (5.9), it is possible to compare the results obtained here with those of Budal (1977) for an infinite row of point absorbers. For $KL < \pi$,

$$q = KL,$$

in agreement with Budal (1977, equation (41) with $\gamma = 0$; note his d is equal to $2L$ here). As q depends on $E_{\max}$ given by (5.9) it is discontinuous whenever $KL = p\pi$ ($p = 1, 2, \ldots$). Budal (1977) only gave results for $KL < \pi$, whereas (5.15) together with (5.9) give q for all values of KL.

6. Conclusion

Results have been derived for a body in a canal by use of Green's theorem which are similar to results derived by other authors for two- and three-dimensional problems (see Newman 1976). These results have been applied to the particular problem of wave-power absorption by a body in a canal. This situation may be taken as a model of an infinite row of absorbers. It has been shown that, for a symmetric body moving in only one mode, $E_{\max} = \frac{1}{2}$ for $0 < KL < \pi$ and that $E_{\max} \leqslant \frac{1}{2}$ for $KL > \pi$ because of the presence of propagating wave modes of wavenumber other than K.

A thin-ship approximation has been employed to consider in detail the effect of these propagating wave modes on $E_{\max}$. It has been shown that the resonance effects which occur when $KL = p\pi$ ($p = 1, 2, \ldots$) and the appearance of a new propagating wave mode as KL goes from $KL < p\pi$ to $KL > p\pi$ can have a detrimental effect on the maximum efficiency. It has also been shown that, for $KL > \pi$, the spacing between bodies relative to the body size has a crucial effect on the maximum efficiency. Thus if the bodies are small compared to their spacing ($d/L \rightarrow 0$) they become less efficient than when they are close together ($d/L \rightarrow 1$).

One limitation of the analysis given here is that it only models an incident wave whose crests are parallel to the rwo of absorbers. It would be more realistic to consider an incident wave whose crests make some arbitrary angle with the row of absorbers. This possibility clearly needs further study. Budal (1977) has considered this case, but only for 'point' absorbers (both finite and infinite rows) and under the restrictive assumption that all the absorbers move with equal amplitudes. He also only partly optimizes with respect to the phases of motion of the absorbers (see Budal 1977, equation (24)). Evans (1979) and Srokosz (1979) both consider the case of a finite row of 'point' absorbers, with a wave incident upon them from an arbitrary angle, but without Budal's restrictive assumptions.

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Appendix. Point source in a canal

In this appendix a derivation for the velocity potential of a point source in a canal is given. The derivation makes use of Havelock's (1929) wave-maker theory. In order to simplify the problem slightly it is assumed that the source is placed at the centre of the canal. Therefore the solution is symmetric with respect to the plane $z = 0$. The velocity potential for a source displaced from the centre of the canal is also given, but the derivation of this result is omitted as the analysis is cumbersome.

Consider a point source situated at $(0, h, 0)$; that is, at the centre of the canal and at a distance h below the free surface. The velocity potential $\phi(x, y, z)$ will be a singular solution of Laplace's equation (2.2) which satisfies the free surface condition (2.3), zero velocity at great depths (2.4), zero normal velocity on the canal walls (2.5) and

$$\phi(x,y,z) \sim \text{const.}\, r^{-1} \quad \text{as} \quad r \to 0, \tag{A 1}$$

where $r^2 = x^2 + (y-h)^2 + z^2$.

(a) Solution using Havelock's wavenumber theory

On the plane $x = 0$ the velocity is given by

$$\left.\frac{\partial \phi}{\partial x}\right|_{x=0\pm} = \pm U(y,z) \tag{A 2}$$

and for a point source at $(0, h, 0)$ $U(y, z)$ is taken to be

$$U(y,z) = m\delta(y-h)\,\delta(z). \tag{A 3}$$

Here m is a constant (related to the source strength) and δ is the Dirac delta function. Now by substituting (A 2) and (A 3) into the results of §2 (equations (2.16)–(2.23)), which were derived using Havelock's wave-maker theory, the following expression is obtained:

$$\begin{aligned}\phi = -\frac{m}{L}\sum_{n=0}^{\infty} \epsilon_n \cos(\alpha_n z)\Bigg\{&K\exp(-K(y+h))\exp(-(\alpha_n^2-K^2)^{\frac{1}{2}}|x|)(\alpha_n^2-K^2)^{-\frac{1}{2}} \\ &+\frac{1}{\pi}\int_0^\infty \frac{(k\cos ky - K\sin ky)(k\cos kh - K\sin kh)\exp(-(\alpha_n^2+k^2)^{\frac{1}{2}}|x|)\,dk}{(k^2+K^2)(\alpha_n^2+k^2)^{\frac{1}{2}}}.\end{aligned} \tag{A 4}$$

As a check that (A 4) does represent a point source it is necessary to show that it satisfies (A 1).

By using results given by Ursell (1951) and Gradshteyn & Ryzhik (1965, pp. 419, 491 and 978), ϕ may be written as

$$\begin{aligned}\phi = -\frac{m}{2\pi}\Bigg\{&\sum_{n=-\infty}^{\infty} [\{x^2+(y-h)^2+(z-2nL)^2\}^{-\frac{1}{2}} - \{x^2+(y+h)^2+(z-2nL)^2\}^{-\frac{1}{2}}] \\ &+\frac{2}{L}\sum_{n=0}^{\infty}\epsilon_n\cos(\alpha_n z)[\pi K\exp(-K(y+h))\exp(-(\alpha_n^2-K^2)^{\frac{1}{2}}|x|)(\alpha_n^2-K^2)^{-\frac{1}{2}} \\ &+\int_0^\infty \frac{k[k\cos k(y+h) - K\sin k(y+h)]\exp(-(\alpha_n^2+k^2)^{\frac{1}{2}}|x|)}{(\alpha_n^2+k^2)^{\frac{1}{2}}(k^2+K^2)}\,dk\Bigg\},\end{aligned} \tag{A 5}$$

where $\alpha_n = n\pi/L$ (for details see Srokosz 1979). From (A 5) it can be seen that ϕ satisfies (A 1) as $r \to 0$. It can also be seen that for $p\pi < KL < (p+1)\pi$ the term involving $\exp\{-(\alpha_n^2-K^2)^{\frac{1}{2}}|x|\}$ represents a propagating wave for $n = 0, \ldots, p$ (as $\alpha_n^2 - K^2 < 0$). Thus the potential amplitudes of the waves propagated to $x = \pm\infty$ are

$$A_n^{\pm} = \frac{imK\,e^{-Kh}}{2(K^2-\alpha_n^2)^{\frac{1}{2}}} \quad \text{for} \quad p\pi < KL < (p+1)\pi \quad \text{and} \quad n = 0, 1, \ldots, p. \tag{A 6}$$

An alternative method of deriving the velocity potential ϕ is to consider an infinite series of three-dimensional point sources placed at the points $(0, h, \pm 2nL)$ for $n = 0, 1, \ldots$. These represent the source in the canal together with its images in the canal walls. Making use of the velocity potential for a three-dimensional point source (given by Thorne 1953) and summing the series leads to the following result (for details see Srokosz 1979):

$$\phi = \sum_{n=-\infty}^{\infty} [\{x^2 + (y-h)^2 + (z-2nL)^2\}^{-\frac{1}{2}} - \{x^2 + (y+h)^2 + (z-2nL)^2\}^{-\frac{1}{2}}]$$
$$- 2\pi i L^{-1} \exp(-K(y+h)) \sum_{n=0}^{p} \epsilon_n \cos(\alpha_n z) \cos[(K^2-\alpha_n^2)^{\frac{1}{2}} x] K(K^2-\alpha_n^2)^{-\frac{1}{2}}$$
$$- \frac{2}{L} \sum_{n=0}^{\infty} \epsilon_n \cos(\alpha_n z) \int_{\alpha_n}^{z} \frac{k \cos[(k^2-\alpha_n^2)^{\frac{1}{2}} x] \exp(-k(y+h))}{(K-k)(k^2-\alpha_n^2)^{\frac{1}{2}}} dk$$

for $p\pi < KL < (p+1)\pi$. This form of ϕ is equivalent to that given in (A 5) except for a constant multiplicative factor of $(-m/2\pi)$ (for a proof of this see Srokosz 1979).

(*b*) *Source displaced from centre of the canal*

If the source is now displaced from the centre of the canal to the point $(0, h, d)$ a similar analysis to that given above leads to the following expression for the velocity potential,

$$\phi = \sum_{n=-\infty}^{\infty} [\{x^2 + (y-h)^2 + (z-d-4nL)^2\}^{-\frac{1}{2}} - \{x^2 + (y+h)^2 + (z-d-4nL)^2\}^{-\frac{1}{2}}$$
$$+ \{x^2 + (y-h)^2 + (z+d+4nL-2L)^2\}^{-\frac{1}{2}} - \{x^2 + (y+h)^2 + (z+d-4nL-2L)^2\}^{-\frac{1}{2}}]$$
$$- 2\pi i L^{-1} \exp(-K(y+h)) \sum_{n=0}^{t} \epsilon_n \cos(\alpha_n d) \cos(\alpha_n z) \cos[(K^2-\alpha_n^2)^{\frac{1}{2}} x] (K^2-\alpha_n^2)^{-\frac{1}{2}}$$
$$- \frac{2}{L} \sum_{n=0}^{\infty} \epsilon_n \cos(\alpha_n d) \cos(\alpha_n z) \int_{\alpha_n}^{\infty} \frac{k \cos[(k^2-\alpha_n^2)^{\frac{1}{2}} x] \exp(-k(y+h))}{(K-k)(k^2-\alpha_n^2)^{\frac{1}{2}}} dk$$
$$- 4\pi i L^{-1} \exp(-K(y+h)) \sum_{n=0}^{t'} \sin(\beta_n d) \sin(\beta_n z) \cos[(K^2-\beta_n^2)^{\frac{1}{2}} x] (K^2-\beta_n^2)^{-\frac{1}{2}}$$
$$- \frac{4}{L} \sum_{n=0}^{\infty} \sin(\beta_n d) \sin(\beta_n z) \int_{\beta_n}^{\infty} \frac{k \cos[(k^2-\beta_n^2)^{\frac{1}{2}} x] \exp(-k(y+h))}{(K-k)(k^2-\beta_n^2)^{\frac{1}{2}}} dk,$$

for $\frac{1}{2}p\pi < KL < \frac{1}{2}(p+1)\pi$. Here $\beta_n = (2n+1)/2L$. There are two cases

(*a*) $p = 2m, \quad t = m, \quad t' = m-1,$

(*b*) $p = 2m+1, \quad t = m, \quad t' = m$

where m may take the values $0, 1, \ldots$; this shows that the number of propagated wave modes depends crucially on the value of KL.

(*c*) *Application to point absorbers*

In the above the velocity potential for a point source in a canal has been derived: it is now possible to use this point source to represent a point absorber in a canal. As noted earlier in §5, Budal's (1977) assumptions for point absorbers are equivalent to using a point source to model the absorber. Thus, by substituting (A 6) into (4.10) it is possible to derive (5.9) as the expression for the maximum efficiency of a point absorber in a canal. This may then be compared to Budal's (1977) results for an infinite row of point absorbers (see §5) and also provide a check on the limit $d/L \to 0$ for the two types of absorber considered in §5.

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